Day 16

Kinematics of Wheeled Robots

Differential Drive

- velocity constraint defines the wheel ground velocities

$$v_r = \omega(R + \frac{\ell}{2})$$

$$v_\ell = \omega(R - \frac{\ell}{2})$$

- given the wheel ground velocities

$$R = \frac{\ell}{2}\frac{(v_r + v_\ell)}{(v_r - v_\ell)}$$

$$\omega = \frac{(v_r - v_\ell)}{\ell}$$

Forward Kinematics

- for a robot starting with pose $[0\ \ 0\ \ 0]^T$ moving with velocity $V(t)$ in a direction $\theta(t)$:

$$x(t) = \int_0^t V(t)\cos(\theta(t))\,dt$$

$$y(t) = \int_0^t V(t)\sin(\theta(t))\,dt$$

$$\theta(t) = \int_0^t \omega(t)\,dt$$

Sensitivity to Wheel Velocity

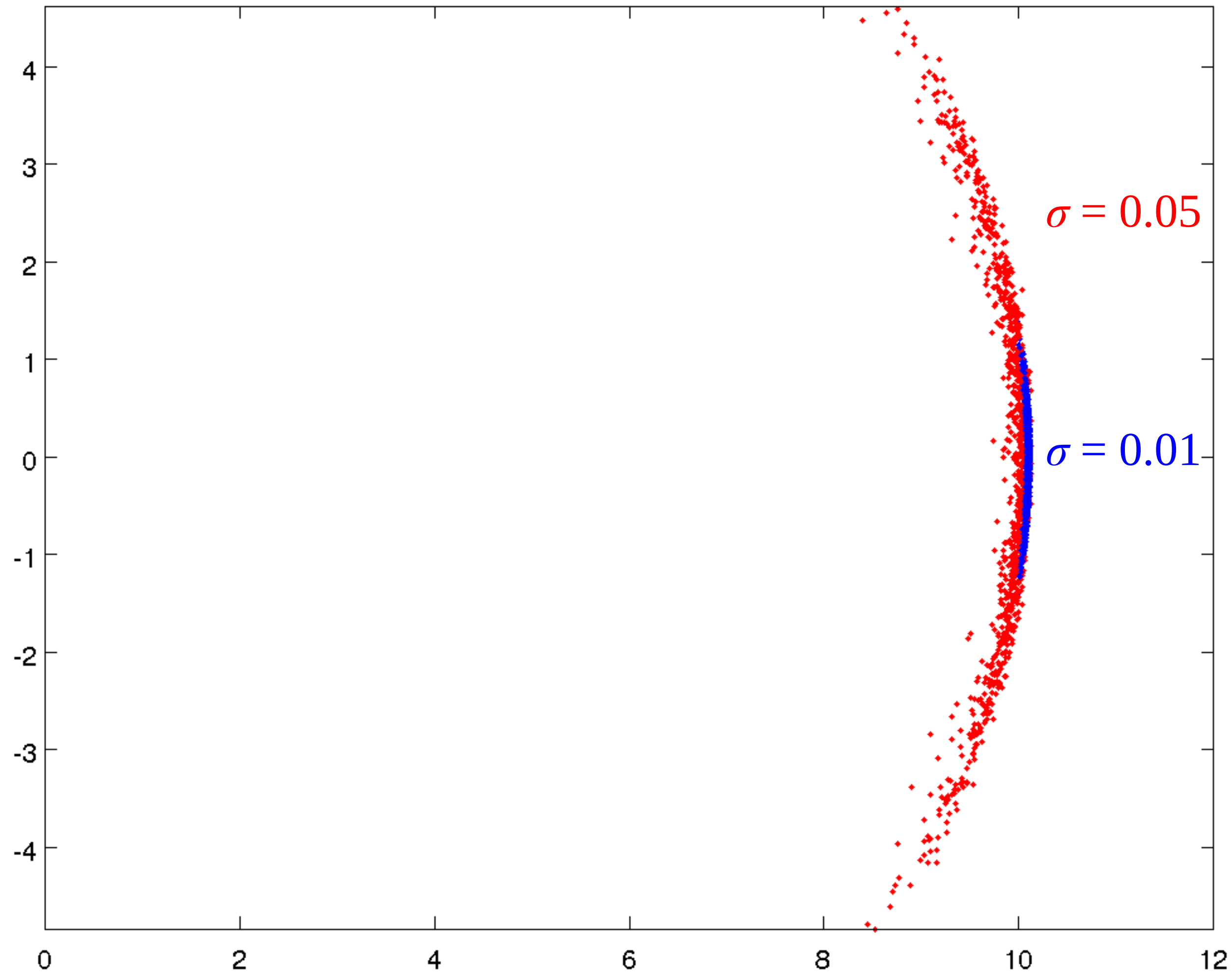
$$v_r(t) = 1 + \mathcal{N}(0, \sigma^2)$$

$$v_\ell(t) = 1 + \mathcal{N}(0, \sigma^2)$$

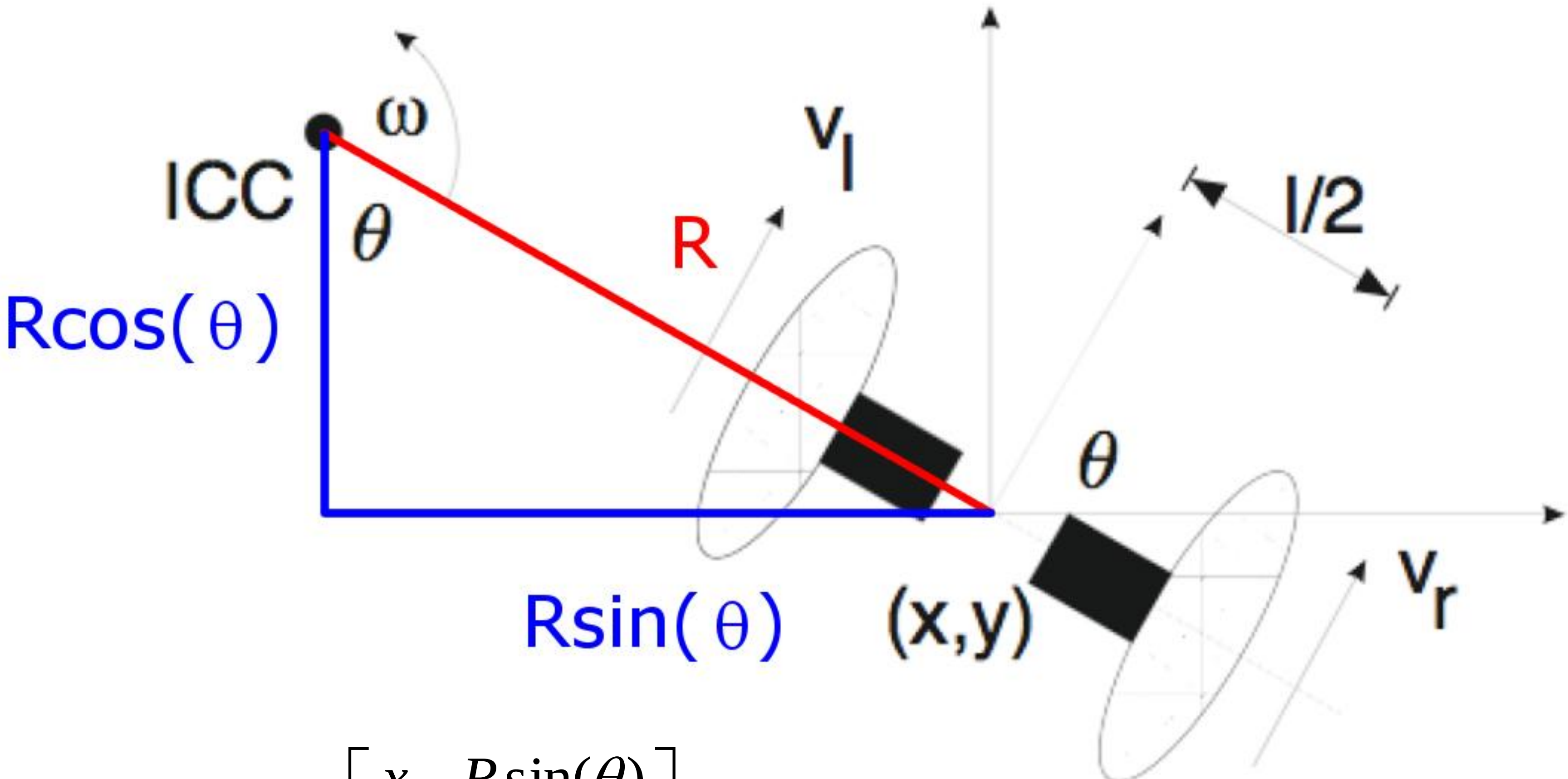
$$\theta(0) = 0$$

$$t = 0 \ldots 10$$

$$\ell = 0.2$$



Correction: ICC for Differential Drive



$$ICC = \begin{bmatrix} x - R\sin(\theta) \\ y + R\cos(\theta) \end{bmatrix}$$

Forward Kinematics

- for differential drive with

$$v_r(t) = v_r, \quad v_\ell(t) = v_\ell, \quad v_r \neq v_\ell, \quad x(0) = y(0) = \theta(0) = 0$$

$$\begin{aligned} x(t) &= \frac{1}{2}\int_0^t (v_r(t) + v_\ell(t))\cos(\theta(t))\,dt \\ &= \frac{\ell}{2}\frac{v_r + v_\ell}{v_r - v_\ell}\sin\left(\frac{t}{\ell}(v_r - v_\ell)\right) \end{aligned}$$

- is this really the same as

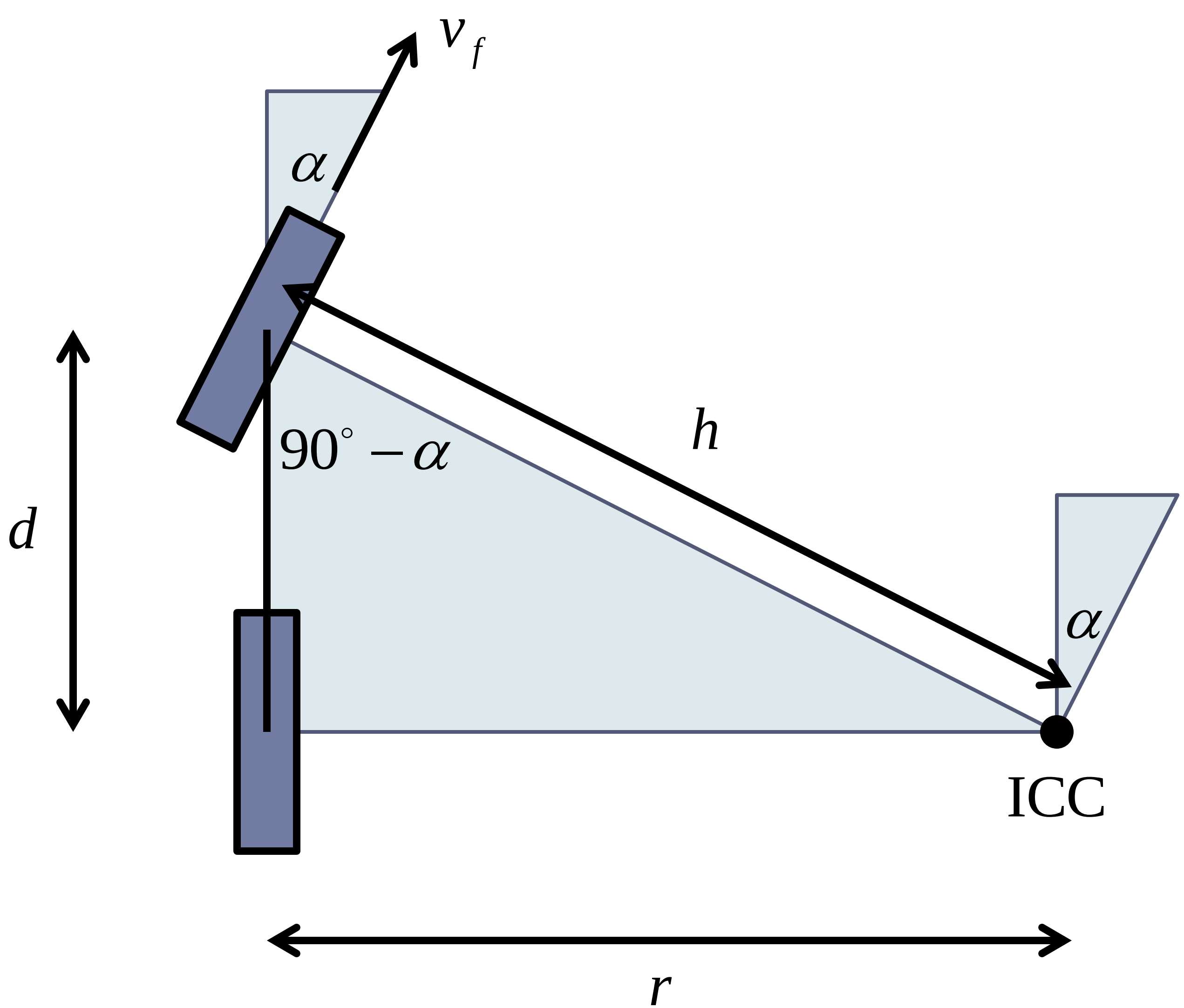
$$\begin{bmatrix} x(t+\delta t) \\ y(t+\delta t) \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) \\ \sin(\omega\delta t) & \cos(\omega\delta t) \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \end{bmatrix}$$

Tracked Vehicles

- similar to differential drive but relies on ground slip or skid to change direction
  - kinematics poorly determined by motion of treads

http://en.wikipedia.org/wiki/File:Tucker-Kitten-Variants.jpg

Steered Wheels: Bicycle



Steered Wheels: Bicycle

- important to remember the assumptions in the kinematic model
  - smooth rolling motion in the plane
- does not capture all possible motions
  - http://www.youtube.com/watch?v=Cj6ho1-G6tw&NR=1#t=0m25s

Mecanum Wheel

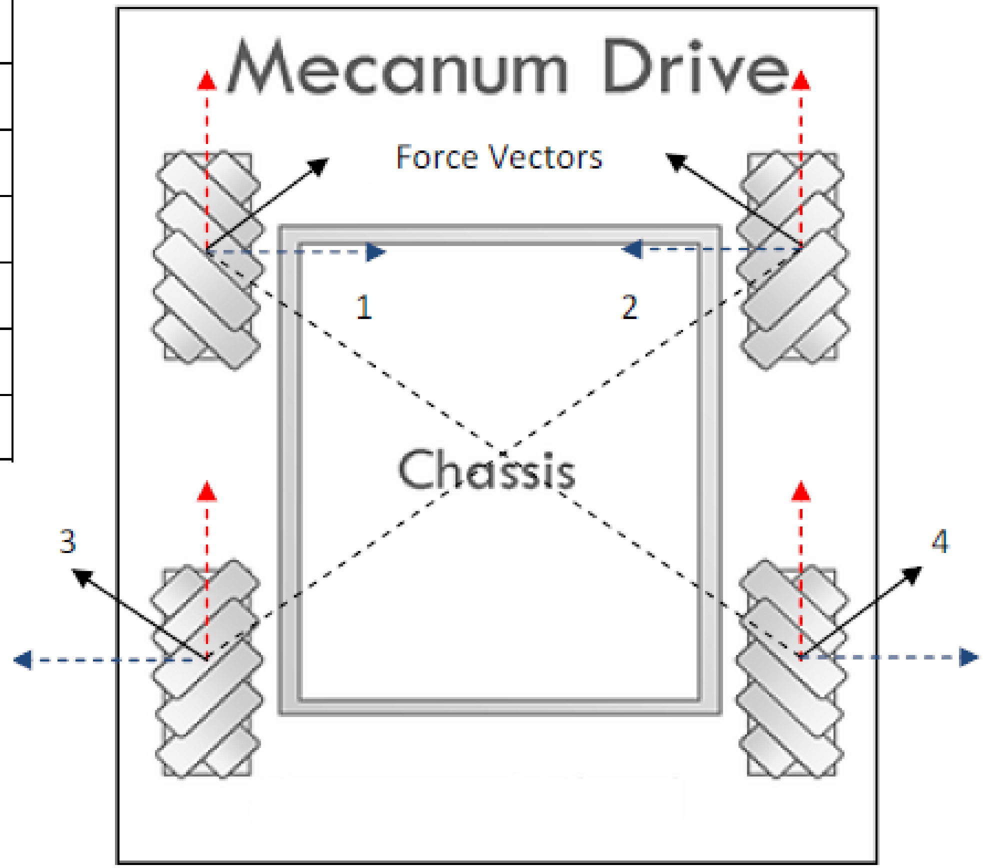
- a normal wheel with rollers mounted on the circumference

http://blog.makezine.com/archive/2010/04/3d-printable-mecanum-wheel.html

- http://www.youtube.com/watch?v=Cee1UZN0p98&feature=player_embedded

Mecanum Wheel

Direction of Movement	Wheel Actuation
Forward	All wheels forward same speed
Reverse	All wheels backward same speed
Right Shift	Wheels 1, 4 forward; 2, 3 backward
Left Shift	Wheels 2, 3 forward; 1, 4 backward
CW Turn	Wheels 1, 3 forward; 2, 4 backward
CCW Turn	Wheels 2, 4 forward; 1, 3 backward



To the right: This is a top view looking down on the drive platform. Wheels in Positions 1, 4 should make X- pattern with Wheels 2, 3. If not set up like shown, wheels will not operate correctly.

AndyMark Mecanum wheel specification sheet
http://d1pytrrjwm20z9.cloudfront.net/MecanumWheelSpecSheet.pdf

Dead Reckoning

- the forward kinematics model we have used computes the pose of the robot based on the previous pose and velocity of the robot
  - there is no reference to the environment external to the robot
- because new poses are computed based only on previous poses, any errors in the pose accumulate over time
  - the lack of external measurements prevents us from correcting the accumulated errors